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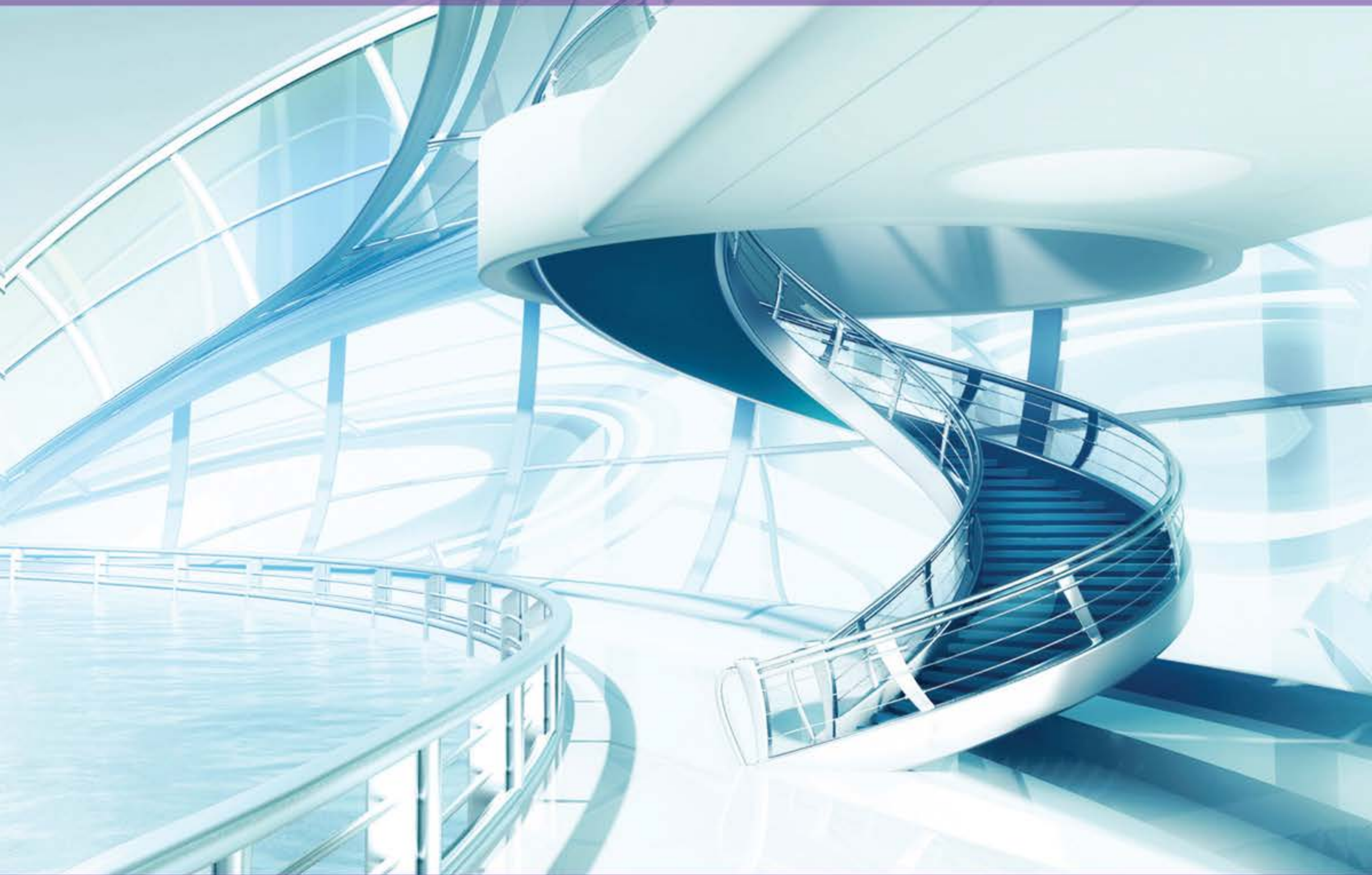

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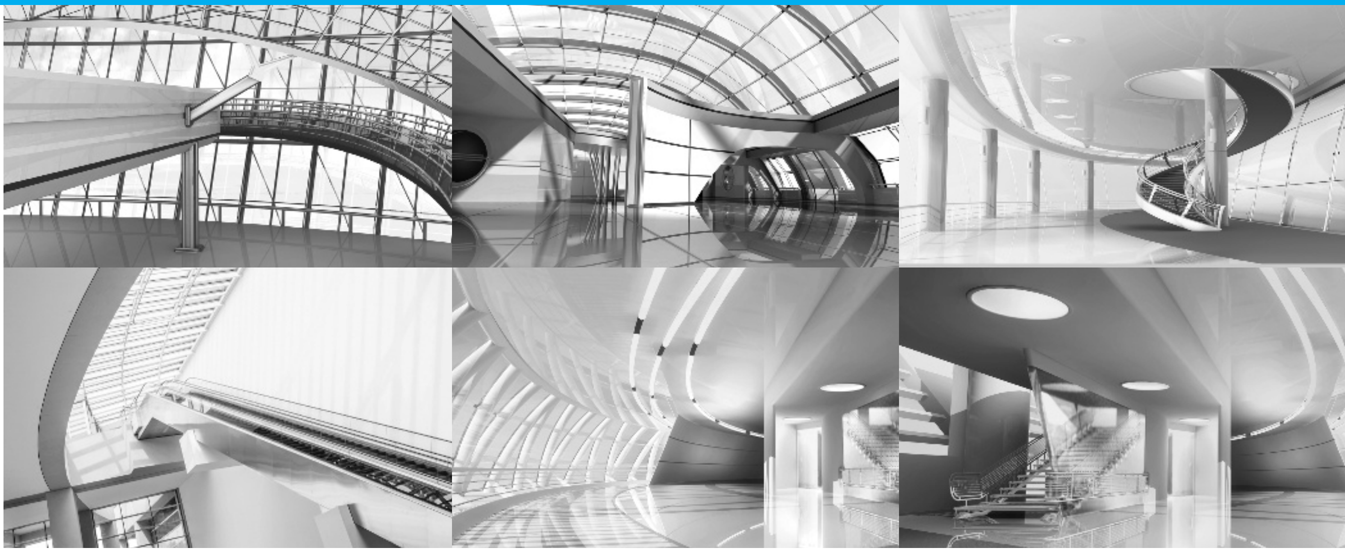

Quantitative Methods for Business 13e

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***Quantitative Methods for Business*, Thirteenth Edition**

David R. Anderson, Dennis J. Sweeney, Thomas A. Williams, Jeffrey D. Camm, James J. Cochran, Michael J. Fry, Jeffrey W. Ohlmann

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To My Children
Krista, Justin, Mark, and Colleen
DRA

To My Children
Mark, Linda, Brad, Tim, Scott, and Lisa
DJS

To My Children
Cathy, David, and Kristin
TAW

To My Family
Karen, Jennifer, Stephanie, and Allison
JDC

To My Wife
Teresa
JJC

To My Family
Nicole and Ian
MJF

To My Family
Amie and Willa
JWO

Brief Contents

Contents

Preface

The purpose of this thirteenth edition, as with previous editions, is to provide undergraduate and graduate students with a conceptual understanding of the role that quantitative methods play in the decision-making process. The text describes the many quantitative methods developed over the years, explains how they work, and shows how the decision maker can apply and interpret them.

This book is applications-oriented and uses our problem scenario approach to gently introduce quantitative material. In each chapter, a problem is described in conjunction with the quantitative procedure being introduced. Development of the quantitative technique or model includes applying it to the problem to generate a solution or recommendation. This approach can help to motivate the student by demonstrating not only how the procedure works, but also how it contributes to the decision-making process.

The mathematical prerequisite for this text is an algebra course. The two chapters on probability and probability distributions will provide the necessary background for the use of probability in subsequent chapters. Throughout the text we use generally accepted notation for the topic being covered. As a result, students who pursue study beyond the level of this text will generally experience little difficulty reading more advanced material. To also assist in further study, a bibliography is included at the end of this book.

CHANGES IN THE THIRTEENTH EDITION

We are very excited about the changes in the thirteenth edition of *Quantitative Methods for Business*, and want to tell you about some of the changes we have made and why.

Updated Chapter 16: Simulation

The most substantial content change in this latest edition involves the coverage of simulation. We maintain an intuitive introduction by continuing to use the concepts of best-, worst-, and base-case scenarios, but we have added a more elaborate treatment of uncertainty by using Microsoft Excel to develop spreadsheet simulation models. Within the chapter, we explain how to construct a spreadsheet simulation model using only native Excel functionality. In the chapter appendix, we describe how an Excel add-in, Analytic Solver Platform, facilitates more sophisticated simulation analyses. This new appendix on Analytic Solver Platform replaces the previous edition's coverage of Crystal Ball, which we no longer pair with our textbook. Nine new problems are introduced, and several others have been updated to reflect the new simulation coverage.

Other Content Changes

A variety of other changes have been made throughout the text in response to user suggestions. The most prominent of these include a new section on variability in project management in Chapter 13, new Appendix A coverage of data tables and Goal Seek functionality in Excel 2013, and adjustment of forecasting notation in Chapter 6. The software previously used to create decision trees in the Chapter 4 appendix, TreePlan, has now been incorporated into the Excel add-in Analytic Solver Platform, and we have updated Chapter 4 accordingly.

New Q.M. in Action, Cases, and Problems

Q.M. in Action is the name of the short summaries that describe how the quantitative methods being covered in the chapter have been used in practice. In this edition, you will find numerous Q.M. in Action vignettes, cases, and homework problems. We have updated many of these Q.M. in Actions to provide more recent examples. In all, we have added 15 new Q.M. in Actions.

The end of each chapter of this book contains cases for students. The cases are more in-depth and often more open-ended than the end-of-chapter homework problems. We have added three new cases to this edition: one on linear programming applications in Chapter 9, one on distribution and network models in Chapter 10, and one on integer programming in Chapter 11. Solutions to all cases are available to instructors.

We have added more than 35 new homework problems to this edition. Many other homework problems have been updated to provide more timely references.

FEATURES AND PEDAGOGY

We continued many of the features that appeared in previous editions. Some of the important ones are noted here.

- Annotations: Annotations that highlight key points and provide additional insights for the student are a continuing feature of this edition. These annotations, which appear in the margins, are designed to provide emphasis and enhance understanding of the terms and concepts presented in the text.
- Notes and Comments: We provide Notes and Comments at the end of many sections to give the student additional insights about the methodology being discussed and its application. These insights include warnings about or limitations of the methodology, recommendations for application, brief descriptions of additional technical considerations, and other matters.
- Self-Test Exercises: Certain exercises are identified as self-test exercises. Completely worked-out solutions for these exercises are provided in Appendix G, entitled Self-Test Solutions and Answers to Even-Numbered Problems, located at the end of the book. Students can attempt the self-test problems and immediately check the solutions to evaluate their understanding of the concepts presented in the chapter. At the request of professors using our textbooks, we now provide the answers to even-numbered problems in this same appendix.
- Q.M. in Action: These articles are presented throughout the text and provide a summary of an application of quantitative methods found in business today. Adaptations of materials from the popular press, academic journals such as *Interfaces*, and write-ups provided by practitioners provide the basis for the applications in this feature.

ANCILLARY LEARNING AND TEACHING MATERIALS

For Students

Print and online resources are available to help the student work more efficiently as well as learn how to use Excel.

- LINGO: The student version of LINGO 14.0 software is available for download at no additional cost to students who purchase a new text, through a link on the student companion site.

- Analytic Solver Platform: An educational version of the latest version of the Analytic Solver Platform software is available at no cost with a new text.

For Instructors

Instructor ancillaries are now provided on the website. Included in this convenient format are the following:

- Solutions Manual: The Solutions Manual, prepared by the authors, includes solutions for all problems in the text.
- Solutions to Case Problems: Also prepared by the authors, it contains solutions to all case problems presented in the text.
- PowerPoint Presentation Slides: Prepared by John Loucks of St. Edwards University, the presentation slides contain a teaching outline that incorporates graphics to help instructors create even more stimulating lectures. The slides may be adapted using PowerPoint software to facilitate classroom use.
- Test Bank: Also prepared by John Loucks, the Test Bank in Microsoft Word files includes multiple choice, true/false, short-answer questions, and problems for each chapter.

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COURSE OUTLINE FLEXIBILITY

The text provides instructors with substantial flexibility in selecting topics to meet specific course needs. Although many variations are possible, the single-semester and single-quarter outlines that follow are illustrative of the options available.

Suggested One-Semester Course Outline

Introduction (Chapter 1)
Probability Concepts (Chapters 2 and 3)
Decision Analysis (Chapters 4 and 5)
Forecasting (Chapter 6)
Linear Programming (Chapters 7, 8, and 9)
Distribution and Network Models (Chapter 10)
Integer Linear Programming (Chapter 11)
Advanced Optimization Applications (Chapter 12)
Project Scheduling: PERT/CPM (Chapter 13)
Simulation (Chapter 15)

Suggested One-Quarter Course Outline

Introduction (Chapter 1)
Decision Analysis (Chapters 4 and 5)
Linear Programming (Chapters 7, 8, and 9)
Distribution and Network Models (Chapter 10)
Integer Linear Programming (Chapter 11)

Advanced Optimization Applications (Chapter 12)
Project Scheduling: PERT/CPM (Chapter 13)
Simulation (Chapter 15)

Many other possibilities exist for one-term courses, depending on the time available, course objectives, and backgrounds of the students.

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Michael J. Fry
Jeffrey W. Ohlmann

CHAPTER 1

Introduction

CONTENTS

This book is concerned with the use of quantitative methods to assist in decision making. It emphasizes not the methods themselves, but rather how they can contribute to better decisions. A variety of names exists for the body of knowledge involving quantitative approaches to decision making. Today, the terms most commonly used—*management science* (MS), *operations research* (OR), *decision science*, and *business analytics*—are often used interchangeably.

The scientific management revolution of the early 1900s, initiated by Frederic W. Taylor, provided the foundation for the use of quantitative methods in management. However, modern research in the use of quantitative methods in decision making, for the most part, originated during the World War II period. At that time, teams of people with diverse specialties (e.g., mathematicians, engineers, and behavioral scientists) were formed to deal with strategic and tactical problems faced by the military. After the war, many of these team members continued their research into quantitative approaches to decision making.

Two developments that occurred during the post–World War II period led to the growth and use of quantitative methods in nonmilitary applications. First, continued research resulted in numerous methodological developments. Arguably the most notable of these developments was the discovery by George Dantzig, in 1947, of the simplex method for solving linear programming problems. At the same time these methodological developments were taking place, digital computers prompted a virtual explosion in computing power. Computers enabled practitioners to use the methodological advances to solve a large variety of problems. The computer technology explosion continues, and personal computers can now be used to solve problems larger than those solved on mainframe computers in the 1990s.

To reinforce the applied nature of the text and to provide a better understanding of the variety of applications in which *quantitative methods* (Q.M.) have been used successfully, Q.M. in Action articles are presented throughout the text. Each Q.M. in Action article summarizes an application of quantitative methods in practice. The first Q.M. in Action, Revenue Management at AT&T Park, describes one of the most important applications of quantitative methods in the sports and entertainment industry.

Q.M. *in* ACTION

*REVENUE MANAGEMENT AT AT&T PARK**

Imagine the difficult position Russ Stanley, Vice President of Ticket Services for the San Francisco Giants, found himself facing late in the 2010 baseball season. Prior to the season, his organization had adopted a dynamic approach to pricing its tickets similar to the model successfully pioneered by Thomas M. Cook and his operations research group at American Airlines. Stanley desparately wanted the Giants to clinch a playoff birth, but he didn't want the team to do so *too quickly*.

When dynamically pricing a good or service, an organization regularly reviews supply and demand of the product and uses operations research to determine if the price should be changed to reflect these conditions. As the scheduled takeoff date for a flight nears, the cost of a ticket increases if seats for the flight are relatively scarce. On the other hand, the airline discounts tickets for an approaching flight with relatively few ticketed passengers. Through the use of optimization to dynamically set ticket prices, American Airlines generates nearly $1 billion annually in incremental revenue.

The management team of the San Francisco Giants recognized similarities between their primary product (tickets to home games) and the primary product sold by airlines (tickets for flights) and adopted a similar revenue management system. If a particular Giants' game is appealing to fans, tickets sell quickly and demand far

(*continued*)

*Based on Peter Horner, "The Sabre Story," *OR/MS Today* (June 2000); Ken Belson, "Baseball Tickets Too Much? Check Back Tomorrow," *New York Times.com* (May 18, 2009); and Rob Gloster, "Giants Quadruple Price of Cheap Seats as Playoffs Drive Demand," *Bloomberg Businessweek* (September 30, 2010).

exceeds supply as the date of the game approaches; under these conditions fans will be willing to pay more and the Giants charge a premium for the ticket. Similarly, tickets for less attractive games are discounted to reflect relatively low demand by fans. This is why Stanley found himself in a quandary at the end of the 2010 baseball season. The Giants were in the middle of a tight pennant race with the San Diego Padres that effectively increased demand for tickets to Giants' games, and the team was actually scheduled to play the Padres in San Fransisco for the last three games of the season. While Stanley certainly wanted his club to win its division and reach the Major League Baseball playoffs, he also recognized that his team's revenues would be greatly enhanced if it didn't qualify for the playoffs until the last day of the season. "I guess financially it is better to go all the way down to the last game," Stanley said in a late season interview. "Our hearts are in our stomachs; we're pacing watching these games."

Does revenue management and operations research work? Today, virtually every airline uses some sort of revenue-management system, and the cruise, hotel, and car rental industries also now apply revenue-management methods. As for the Giants, Stanley said dynamic pricing provided a 7 to 8% increase in revenue per seat for Giants' home games during the 2010 season. Coincidentally, the Giants did win the National League West division on the last day of the season and ultimately won the World Series. Several professional sports franchises are now looking to the Giants' example and considering implementation of similar dynamic ticket-pricing systems.

1.1 Problem Solving and Decision Making

Problem solving can be defined as the process of identifying a difference between the actual and the desired state of affairs and then taking action to resolve this difference. For problems important enough to justify the time and effort of careful analysis, the problem-solving process involves the following seven steps:

1. Identify and define the problem.
2. Determine the set of alternative solutions.
3. Determine the criterion or criteria that will be used to evaluate the alternatives.
4. Evaluate the alternatives.
5. Choose an alternative.
6. Implement the selected alternative.
7. Evaluate the results to determine whether a satisfactory solution has been obtained.

Decision making is the term generally associated with the first five steps of the problem-solving process. Thus, the first step of decision making is to identify and define the problem. Decision making ends with the choosing of an alternative, which is the act of making the decision.

Let us consider the following example of the decision-making process. For the moment, assume you are currently unemployed and that you would like a position that will lead to a satisfying career. Suppose your job search results in offers from companies in Rochester, New York; Dallas, Texas; Greensboro, North Carolina; and Pittsburgh, Pennsylvania. Further suppose that it is unrealistic for you to decline all of these offers. Thus, the alternatives for your decision problem can be stated as follows:

1. Accept the position in Rochester.
2. Accept the position in Dallas.
3. Accept the position in Greensboro.
4. Accept the position in Pittsburgh.

The next step of the problem-solving process involves determining the criteria that will be used to evaluate the four alternatives. Obviously, the starting salary is a factor of some importance. If salary were the only criterion important to you, the alternative selected as "best" would be the one with the highest starting salary. Problems in which the objective is to find the best solution with respect to one criterion are referred to as **single-criterion decision problems**.

Suppose that you also conclude that the potential for advancement and the location of the job are two other criteria of major importance. Thus, the three criteria in your decision problem are starting salary, potential for advancement, and location. Problems that involve more than one criterion are referred to as **multicriteria decision problems**.

The next step of the decision-making process is to evaluate each of the alternatives with respect to each criterion. For example, evaluating each alternative relative to the starting salary criterion is done simply by recording the starting salary for each job alternative. However, evaluating each alternative with respect to the potential for advancement and the location of the job is more difficult because these evaluations are based primarily on subjective factors that are often difficult to quantify. Suppose for now that you decide to measure potential for advancement and job location by rating each of these criteria as poor, fair, average, good, or excellent. The data you compile are shown in Table 1.1.

You are now ready to make a choice from the available alternatives. What makes this choice phase so difficult is that the criteria are probably not all equally important, and no one alternative is "best" with regard to all criteria. When faced with a multicriteria decision problem, the third step in the decision-making process often includes an assessment of the relative importance of the criteria. Although we will present a method for dealing with situations like this one later in the text, for now let us suppose that after a careful evaluation of the data in Table 1.1, you decide to select alternative 3. Alternative 3 is thus referred to as the **decision**.

At this point in time, the decision-making process is complete. In summary, we see that this process involves five steps:

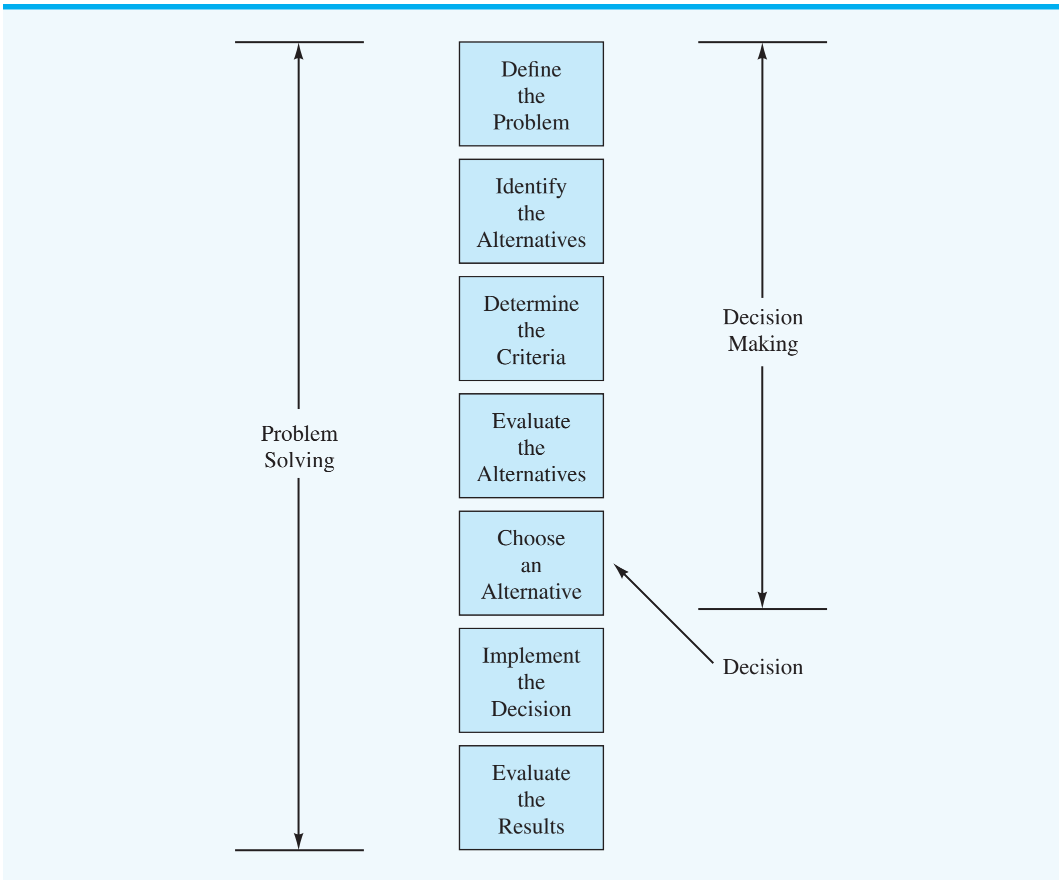
1. Define the problem.
2. Identify the alternatives.
3. Determine the criteria.
4. Evaluate the alternatives.
5. Choose an alternative.

Note that missing from this list are the last two steps in the problem-solving process: implementing the selected alternative and evaluating the results to determine whether a satisfactory solution has been obtained. This omission is not meant to diminish the importance

TABLE 1.1 DATA FOR THE JOB EVALUATION DECISION-MAKING PROBLEM

Alternative	Starting Salary	Potential for Advancement	Job Location
1. Rochester	$48,500	Average	Average
2. Dallas	$46,000	Excellent	Good
3. Greensboro	$46,000	Good	Excellent
4. Pittsburgh	$47,000	Average	Good

FIGURE 1.1 THE RELATIONSHIP BETWEEN PROBLEM SOLVING AND DECISION MAKING



of each of these activities, but to emphasize the more limited scope of the term *decision making* as compared to the term *problem solving*. Figure 1.1 summarizes the relationship between these two concepts.

1.2 Quantitative Analysis and Decision Making

Consider the flowchart presented in Figure 1.2. Note that we combined the first three steps of the decision-making process under the heading of "Structuring the Problem" and the latter two steps under the heading "Analyzing the Problem." Let us now consider in greater detail how to carry out the activities that make up the decision-making process.

Figure 1.3 shows that the analysis phase of the decision-making process may take two basic forms: qualitative and quantitative. Qualitative analysis is based primarily on the manager's judgment and experience; it includes the manager's intuitive "feel" for the problem and is more an art than a science. If the manager has had experience with similar problems, or if the problem is relatively simple, heavy emphasis may be placed upon a qualitative analysis. However, if the manager has had little experience with similar problems, or if the problem

FIGURE 1.2 A SUBCLASSIFICATION OF THE DECISION-MAKING PROCESS

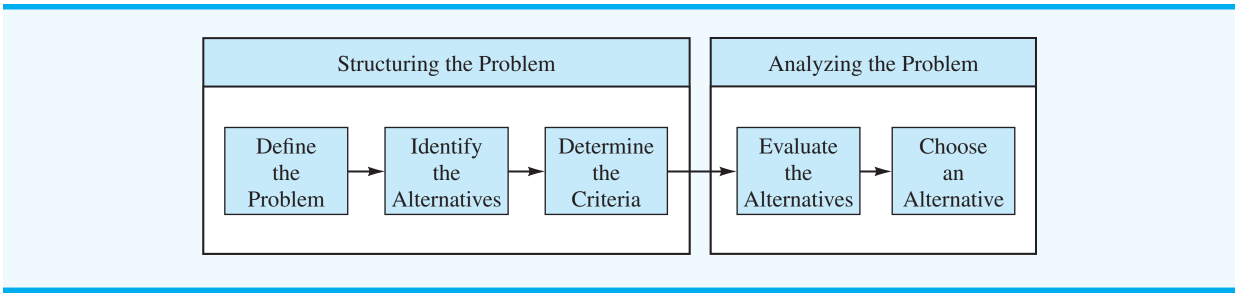
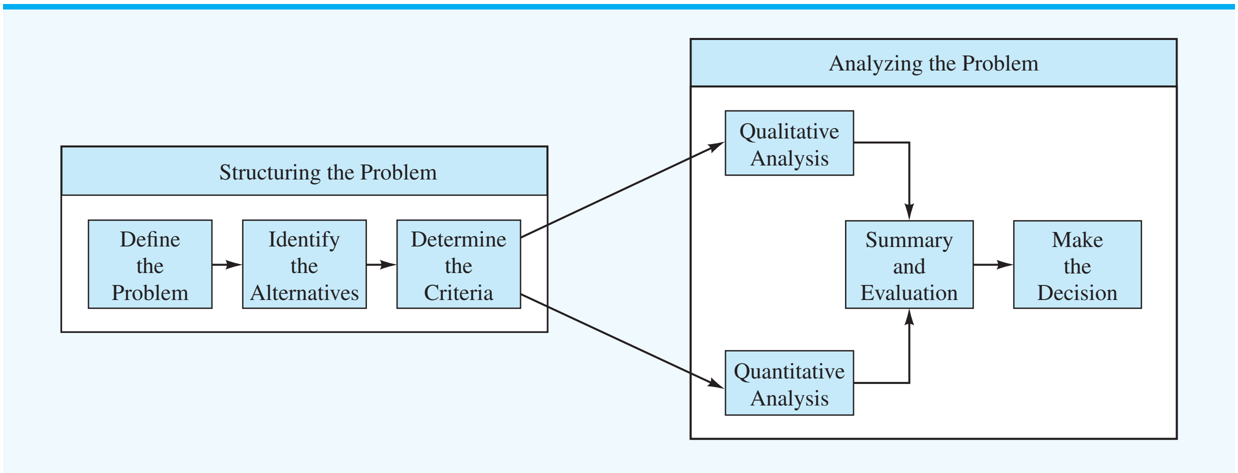


FIGURE 1.3 THE ROLE OF QUALITATIVE AND QUANTITATIVE ANALYSIS



is sufficiently complex, then a quantitative analysis of the problem can be an especially important consideration in the manager's final decision.

Quantitative methods are especially helpful with large, complex problems. For example, in the coordination of the thousands of tasks associated with landing the Apollo 11 safely on the moon, quantitative techniques helped to ensure that more than 300,000 pieces of work performed by more than 400,000 people were integrated smoothly.

When using a quantitative approach, an analyst will concentrate on the quantitative facts or data associated with the problem and develop mathematical expressions that describe the objectives, constraints, and other relationships that exist in the problem. Then, by using one or more mathematical methods, the analyst will make a recommendation based on the quantitative aspects of the problem.

Although skills in the qualitative approach are inherent in the manager and usually increase with experience, the skills of the quantitative approach can be learned only by studying the assumptions and methods of management science. A manager can increase decision-making effectiveness by learning more about quantitative methodology and by better understanding its contribution to the decision-making process. A manager who is knowledgeable in quantitative decision-making procedures is in a much better position to compare and evaluate the qualitative and quantitative sources of recommendations and ultimately to combine the two sources to make the best possible decision.

The box in Figure 1.3 entitled "Quantitative Analysis" encompasses most of the subject matter of this text. We will consider a managerial problem, introduce the appropriate quantitative methodology, and then develop the recommended decision.

Some of the reasons why a quantitative approach might be used in the decision-making process include the following:

1. The problem is complex, and the manager cannot develop a good solution without the aid of quantitative analysis.
2. The problem is critical (e.g., a great deal of money is involved), and the manager desires a thorough analysis before making a decision.
3. The problem is new, and the manager has no previous experience from which to draw.
4. The problem is repetitive, and the manager saves time and effort by relying on quantitative procedures to automate routine decision recommendations.

1.3 Quantitative Analysis

From Figure 1.3 we see that quantitative analysis begins once the problem has been structured. It usually takes imagination, teamwork, and considerable effort to transform a rather general problem description into a well-defined problem that can be approached via quantitative analysis. It is important to involve the stakeholders (the decision maker, users of results, etc.) in the process of structuring the problem to improve the likelihood that the ensuing quantitative analysis will make an important contribution to the decision-making process. When those familiar with the problem agree that it has been adequately structured, work can begin on developing a model to represent the problem mathematically. Solution procedures can then be employed to find the best solution for the model. This best solution for the model then becomes a recommendation to the decision maker. The process of developing and solving models is the essence of the quantitative analysis process.

Model Development

Models are representations of real objects or situations and can be presented in various forms. For example, a scale model of an airplane is a representation of a real airplane. Similarly, a child's toy truck is a model of a real truck. The model airplane and toy truck are examples of models that are physical replicas of real objects. In modeling terminology, physical replicas are referred to as **iconic models**.

A second classification includes models that are physical in form but do not have the same physical appearance as the object being modeled. Such models are referred to as **analog models**. The speedometer of an automobile is an analog model; the position of the needle on the dial represents the speed of the automobile. A thermometer is another analog model representing temperature.

A third classification of models—the type we will primarily be studying—includes representations of a problem by a system of symbols and mathematical relationships or expressions. Such models are referred to as **mathematical models** and are a critical part of any quantitative approach to decision making. For example, the total profit from the sale of a product can be determined by multiplying the profit per unit by the quantity sold. Let x represent the number of units produced and sold, and let P represent the total profit. With a profit of \$10 per unit, the following mathematical model defines the total profit earned by producing and selling x units:

$$P = 10x \quad (1.1)$$

The purpose, or value, of any model is that it enables us to make inferences about the real situation by studying and analyzing the model. For example, an airplane designer might test an iconic model of a new airplane in a wind tunnel to learn about the potential flying characteristics of the full-size airplane. Similarly, a mathematical model may be used to make inferences about how much profit will be earned if a specified quantity of a particular product is sold. According to the mathematical model of equation (1.1), we would expect that selling three units of the product ($x = 3$) would provide a profit of $P = 10(3) = \$30$.

In general, experimenting with models requires less time and is less expensive than experimenting with the real object or situation. One can certainly build and study a model airplane in less time and for less money than it would take to build and study the full-size airplane. Similarly, the mathematical model in equation (1.1) allows a quick identification of profit expectations without requiring the manager to actually produce and sell x units. Models also reduce the risks associated with experimenting with the real situation. In particular, bad designs or bad decisions that cause the model airplane to crash or the mathematical model to project a $10,000 loss can be avoided in the real situation.

Herbert A. Simon, a Nobel Prize winner in economics and an expert in decision making, said that a mathematical model does not have to be exact; it just has to be close enough to provide better results than can be obtained by common sense.

The value of model-based conclusions and decisions depends on how well the model represents the real situation. The more closely the model airplane represents the real airplane, the more accurate will be the conclusions and predictions. Similarly, the more closely the mathematical model represents the company's true profit–volume relationship, the more accurate will be the profit projections.

Because this text deals with quantitative analysis based on mathematical models, let us look more closely at the mathematical modeling process. When initially considering a managerial problem, we usually find that the problem definition phase leads to a specific objective, such as maximization of profit or minimization of cost, and possibly a set of restrictions or **constraints**, which express limitations on resources. The success of the mathematical model and quantitative approach will depend heavily on how accurately the objective and constraints can be expressed in mathematical equations or relationships.

The mathematical expression that defines the quantity to be maximized or minimized is referred to as the **objective function**. For example, suppose x denotes the number of units produced and sold each week, and the firm's objective is to maximize total weekly profit. With a profit of \$10 per unit, the objective function is $10x$. A production capacity constraint would be necessary if, for instance, 5 hours are required to produce each unit and only 40 hours are available per week. The production capacity constraint is given by

$$5x \leq 40 \qquad \textbf{(1.2)}$$

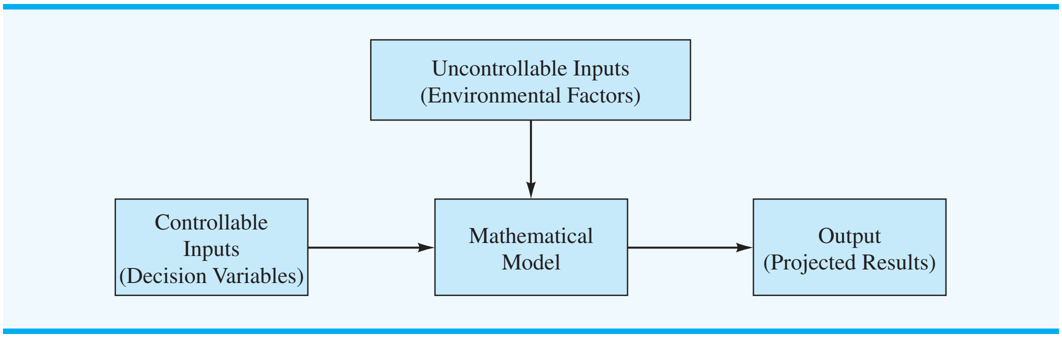
The value of $5x$ is the total time required to produce x units; the symbol $\leq$ indicates that the production time required must be less than or equal to the 40 hours available.

The decision problem or question is the following: How many units of the product should be produced each week to maximize profit? A complete mathematical model for this simple production problem is

$$\begin{aligned} &\text{Maximize} \quad 10x \quad \text{objective function} \\ &\text{subject to (s.t.)} \\ &\left.\begin{aligned} 5x &\leq 40 \\ x &\geq 0 \end{aligned}\right\} \text{constraints} \end{aligned}$$

The $x \geq 0$ constraint requires the production quantity x to be greater than or equal to zero, which simply recognizes the fact that it is not possible to manufacture a negative number

FIGURE 1.4 FLOWCHART OF THE PROCESS OF TRANSFORMING MODEL INPUTS INTO OUTPUT



of units. The optimal solution to this simple model can be easily calculated and is given by $x = 8$, with an associated profit of \$80. This model is an example of a linear programming model. In subsequent chapters we will discuss more complicated mathematical models and learn how to solve them in situations for which the answers are not nearly so obvious.

In the preceding mathematical model, the profit per unit (\$10), the production time per unit (5 hours), and the production capacity (40 hours) are factors not under the control of the manager or decision maker. Such factors, which can affect both the objective function and the constraints, are referred to as **uncontrollable inputs** to the model. Inputs that are controlled or determined by the decision maker are referred to as **controllable inputs** to the model. In the example given, the production quantity x is the controllable input to the model. Controllable inputs are the decision alternatives specified by the manager and thus are also referred to as the **decision variables** of the model.

Once all controllable and uncontrollable inputs are specified, the objective function and constraints can be evaluated and the output of the model determined. In this sense, the output of the model is simply the projection of what would happen if those particular factors and decisions occurred in the real situation. A flowchart of how controllable and uncontrollable inputs are transformed by the mathematical model into output is shown in Figure 1.4. A similar flowchart showing the specific details for the production model is shown in Figure 1.5. Note that we have used "Max" as an abbreviation for maximize.

FIGURE 1.5 FLOWCHART FOR THE PRODUCTION MODEL